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ABSTRACT

This booklet is a collection of activities and games designed to supplement textbook and other instructional materials in an elementary school mathematics program. The selected activities propose to stimulate learning and enhance attitudes. Some provide practice with number facts; others explore various topics such as probability. The emphasis is on the affective domain, and the activities are not designed necessarily for mastery by all students. The purpose is pupil involvement without fear of failure. (LS)

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ACTIVITIES FOR ELEMENTARY SCHOOL MATHEMATICS ENRICHMENT

CURRICULUM RESEARCH
AND DEVELOPMENT CENTER

SCHOOL OF EDUCATION,

INDIANA STATE UNIVERSITY

TERRE HAUTE

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School of Education, Indiana State University

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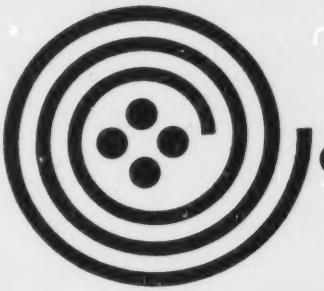

CURRICULUM RESEARCH
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FOREWORD

Textbooks and workbooks do not, in and of themselves, provide a total program in mathematics for elementary school children. What the teacher and children do with all available instructional materials is what really counts. A complete mathematics program for children makes use of selected instructional devices, to complement and supplement the textbook and workbook.

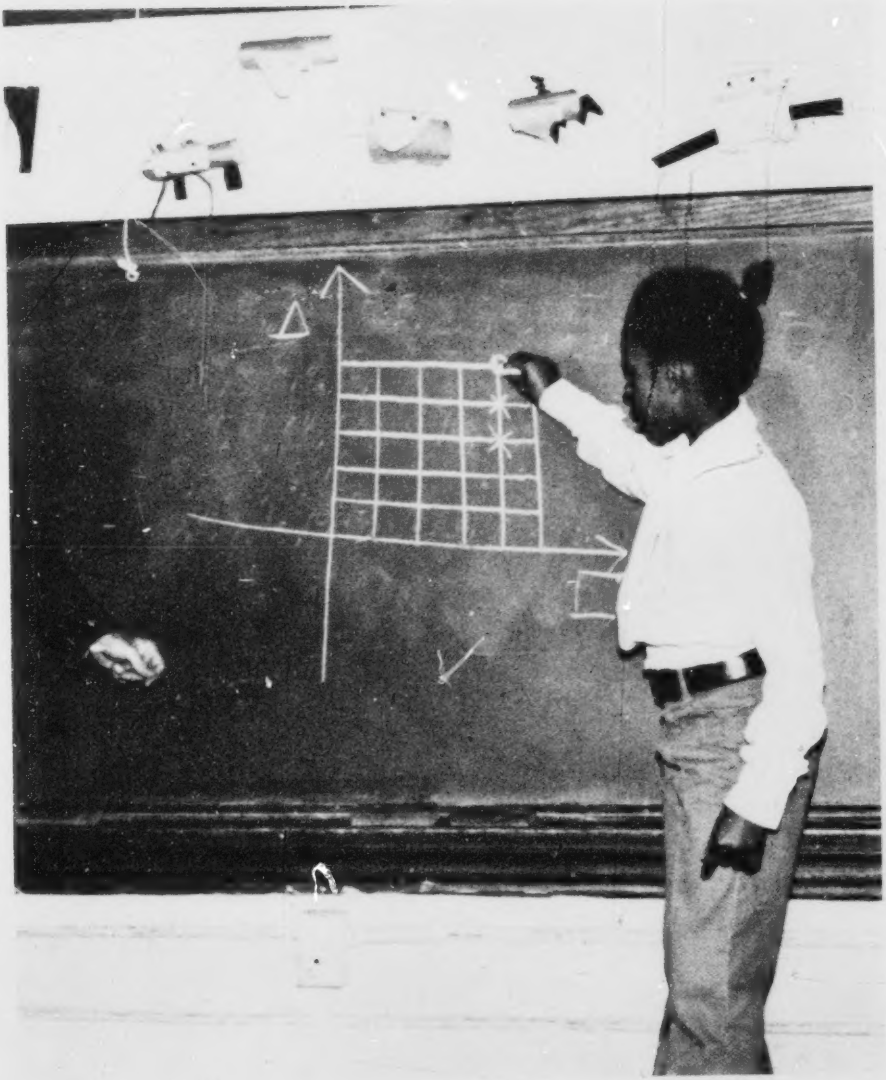
The key word above is the word selected. The authors of this activity book have exercised careful consideration of many possible activities. The ones they have chosen to include in the book are child-tested. Few other activity books available can make this claim.

One of the major outcomes of a modern mathematics program is the development of a positive attitude toward the material being studied. The wise use of these selected activities to supplement the concepts, under study, has been shown to increase interest and enhance favorable attitudes.

There is nothing wrong with the idea of fun and games in mathematics. As indicated they serve a most useful purpose. However, if activities can serve more than one purpose they can complement the program in multiple ways. As the reader looks at these selected activities, I am sure that the authors' attention to the development of rational thought on the part of children is readily apparent.

Every teacher wants to develop a program where children are challenged, in a positive way, and interest is high. There is no doubt that the materials in this book will go a long way toward bringing such a program into reality.

Ronald C. Welch
Professor of Mathematics Education
Indiana University, 1974



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PREFACE

There are two major purposes of this booklet. First of all, teachers of elementary school children are constantly looking for meaningful activities they can use to supplement their ongoing programs. Secondly, children need experiences in which they become actively involved in their learning, enjoy it, and experience success. The materials in this booklet are suggested as activities in mathematics which will assist the elementary teacher in meeting these objectives.

The authors have chosen not to organize the activities in the booklet in the traditional chapter form. It was felt that this would tend to isolate an activity and imply that it was appropriate only for that section (i.e., place value, drill for addition, etc.). Many of the activities can, with minor modification, be used in more than one aspect of elementary school mathematics.

All of the activities contained in this booklet have been used by the authors in elementary classrooms with children. The children have responded with enthusiasm. The authors feel that the teacher who enriches her elementary mathematics program with these activities will have similar results.

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INTRODUCTION

More than a decade has elapsed since elementary schools in the United States first experienced the revolution in "modern" mathematics instruction. During that time, major changes have been made in textbook series, thus having an effect on the teachers using them and, hopefully, the children being instructed by those teachers.

While the majority of concepts and skills stressed today are the same as those in earlier times, it is true that additional topics can be found in the present curriculum. The modern program is based upon sets and the various properties (commutative, associative, etc.). More emphasis is placed upon geometry than was previously the case. Emerging trends indicate that there will soon be additional work in probability, statistics, and the metric system.

Today's mathematics curriculum is not without its critics. During its infancy the movement was attacked for numerous reasons, most notably a "fear" of doing something different, therefore breaking away from tradition. More recently, however, criticisms have been based upon the belief that children are not achieving in mathematics as they should be.

It is true that children need to be able to compute the answers to mathematical problems with accuracy, but other things are of importance also. Rational thinking is a desirable attribute. True mathematical understanding is a logical expectation. It is not enough for a child to know that 18 - 9 = 9. He must be able to understand why this is so.

As well as understanding how to subtract, for example, the child must know when to subtract.

Assume that many of the criticisms of newer mathematics programs for the elementary school are justified. Where does the responsibility rest? Not in the content of the program! The man who drives his car through the back of his garage usually has himself to blame. The automobile has only responded to what the driver directed it to do.

Does this mean that teachers are not of the quality that they once were? Not at all! Is it possible that the newer programs have been dealt with using traditional methods? Possibly, but not to a great degree. Have children of the present generation evolved into a new conglomerate, far different from those of twenty years ago? To some degree, perhaps, but not in such a degree that one would notice significant differences.

There can be no denying the fact that the newer programs in elementary school mathematics resulted in part from a belief on the part of many educators in the United States that a real weakness existed in our school systems. Sputnik I was indeed a shock to this country's central nervous system, better known as the public schools. However, the treatment consisted generally of attacking the cognitive domain by increasing the content to be offered, in addition to making some changes in the content already included.

Could it be that more emphasis in the affective domain could strengthen the programs now in existence? The authors of this booklet feel that the answer is emphatically "yes," and that one method is through enrichment.

Enrichment is basically what its name implies. It consists of learning experiences outside the realm of the normal curriculum, designed to broaden the child academically, either vertically or horizontally. Such activities are for the purpose of pupil involvement, without fear of failure on the part of the pupil who does not grasp the concept(s) involved. The activities are not designed necessarily for mastery. In short, children can enjoy what they are doing without feeling pressures from without to do well.

When children experience success, attitudes toward learning are enhanced. When children want to learn, the teacher's goals for instruction are more easily attained.

The teachers who generally use a textbook with their elementary children, whether they group the students or teach them as a total group, will find that occasional use of selected enrichment activities provides a pleasant break in routine, thereby giving everyone involved a "second wind." Total group enrichment activities can also provide a needed "breather" to those teachers and children involved in the various individualized programs presently in existence.

Although not a panacea, enrichment can help provide children with stimulation while learning and stimulation to learn in other settings. The activities that can be used are limited only by the teacher's creativity and imagination. The rest of this book is devoted to presenting specific activities that teachers may use with elementary school students.

PROBABILITY

The topic of probability is emerging as a necessary component in today's elementary mathematics curriculum. Until recently, it was regarded as being reserved for upper elementary and beyond. Such is not the case. Probability has its applications beginning as early as kindergarten. Following are brief descriptions of some activities one might use with children. No attempt has been made to designate specific grade levels, leaving that decision to the teacher.

Activity A

Place 5 red and 5 white poker chips in a shoebox. Shake the box and then have one child blindly select five chips. Make a record of the selection and then place the chips back in the shoebox. Repeat the activity nine times, allowing a different child to make the selections on each occasion. Then discuss the following:

a) How many different outcomes?

b) What are the most likely outcomes? (3R - 2W or 3W - 2R)

c) What are the least likely outcomes? (5R or 5W)

d) How many chips would have to be drawn from the box to guarantee five of the same color? (9)

Activity B

Take 15 red poker chips and 5 white ones. Place them in a shoebox. Have one child blindly select four of them. Record the selection and replace the chips in the box. Have five other children make similar samples and record those. Tell the pupils that the shoebox contains 20 chips and then discuss the following:

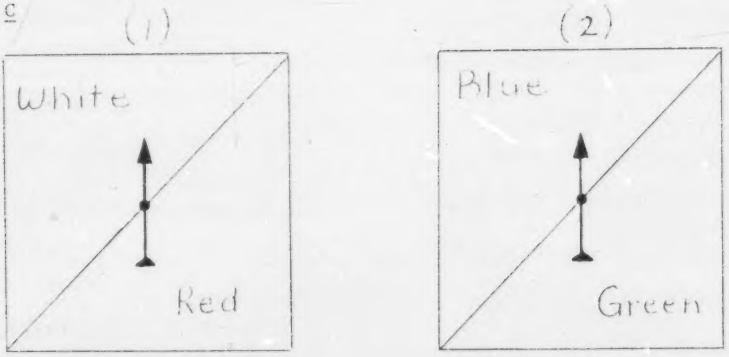
a) The meaning of "taking a sample."

b) What the students feel the ratio of red to white is in the box. (Hopefully, they'll say 3:1.)

c) The total number of red and white chips in the box.

d) The ratio if 10 more red chips were added; the ratio if 10 white rather than 10 red were added. (One might want to verify this by taking additional samples after placing more chips in the box.)

Activity C



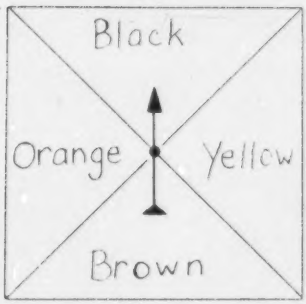
Given two spinners colored as indicated above, discuss the following:

a) On spinner 1, what is the probability of getting a white? (1 chance in 2.) A red? (1 chance in 2.) The same for blue and green on spinner 2? (Yes.)

b) Using both spinners simultaneously, what are the chances of getting a white-blue? (1 in 4.) A red-blue, white-green, or a red-green? (1 in 4.) A blue-green? (Zero, since they are on the same spinner.)

Given a third spinner (page 7), what are the chances of getting any one of the colors? (1 in 4.) Using spinners 1 and 3, what are the chances of getting a white and a yellow? (1 in 8.) Using all three spinners, what chances exist of getting a red-green-black combination? (1 in 16.)

(3)



Activity D

Take two containers. In the first, place 10 black marbles and 5 white ones. In the second container, place 20 black and 5 white. Ask the following questions?

a) If I wanted a black marble, from which container should I draw?

b) Could I get a black marble from either one?

c) Is it possible to get a white marble from either one?

d) Which container would give me the best chance of drawing a white marble?

e) If I drew six marbles from container one, how many black and how many white should there be? (4 and 2.)

f) If I took five marbles from container two, how many should be black? (4.)

Activity E

Give each of the children one die (singular for dice). Have them roll twelve times and record their results on a table as below.

FACE	⚀	⚁	⚂	⚃	⚄	⚅
OCCURRENCES						

The chances of each face appearing is one in six, so it would be projected that in twelve rolls, each face would appear two times. However, since the sample is small, this may not be the case. This can be allowed for by compiling the results of all the children, thus increasing the sample and therefore the chances of getting a more normal distribution.

ODDS AND EVENS

The teacher might decide to let pupils experiment with all or a part of the following activity, depending upon the grade level and mathematical maturity of the children involved. It will be readily apparent that some interesting generalizations can be derived by elementary pupils based upon these activities.

For each of the following, let O = odd; E = even:

E + E = □

E + O = △

E - E = ▽

O - E = △

O + E = □

O + O = △

E + E + E = △

O + O + O = □

O + E + □ + O = E

E + E + E + E + E + O + E + E + E + E = △

E - O = ▽

O - O = □

O - O - O + □ = E + E

E + E = O + □

4 x E = △

3 x E = ▽

E x E = □

O x O = △

E^2 = □

E^3 = ▽

O^2 = △ O^3 = □

CLOCK ARITHMETIC

A good way to begin this type of activity is to tell the pupils that some number sentences are going to be placed upon the chalkboard, all of which are true sentences. Let the sentences be of the following type:

3 + 3 = 6
4 + 5 = 9
7 + 4 = 11
8 + 5 = 1
7 + 9 = 4
9 + 9 = 6
11 + 1 = 12
11 + 11 = 10

Little reaction will be given by members of the class to the first three sentences, but they immediately will become very suspect of the remaining five. When asked if they could think of a number system in which the eight sentences could all be true, the responses will surely be varied. If none are able to figure out the system, the following types of leading questions might be used:

"At 3 hours after 3:00 o'clock, what time is it?"
"At 5 hours after 4:00 o'clock, what time is it?"
"At 4 hours after 7:00 o'clock, what time is it?"
"At 5 hours after 8:00 o'clock, what time is it?"
"At 9 hours after 7:00 o'clock, what time is it?"
"At 9 hours after 9:00 o'clock, what time is it?"
"At 1 hour after 11:00 o'clock, what time is it?"
"At 11 hours after 11:00 o'clock, what time is it?"

Many other problems of this type can then be used. While it is not essential to either the understanding or the enjoyment of the activity, the teacher may decide to inform the class that this is called "modular arithmetic," and that these problems would be the result of working in a modulo 12.

The pupils may wish to build an addition table for modulo 12 and use it to solve simple addition and subtraction problems. While multiplication and division can also be fun in modular arithmetic, the teacher may decide that the level of maturity of the class does not warrant such an exploration. The completed addition table would appear as:

+	1	2	3	4	5	6	7	8	9	10	11	12
1	2	3	4	5	6	7	8	9	10	11	12	1
2	3	4	5	6	7	8	9	10	11	12	1	2
3	4	5	6	7	8	9	10	11	12	1	2	3
4	5	6	7	8	9	10	11	12	1	2	3	4
5	6	7	8	9	10	11	12	1	2	3	4	5
6	7	8	9	10	11	12	1	2	3	4	5	6
7	8	9	10	11	12	1	2	3	4	5	6	7
8	9	10	11	12	1	2	3	4	5	6	7	8
9	10	11	12	1	2	3	4	5	6	7	8	9
10	11	12	1	2	3	4	5	6	7	8	9	10
11	12	1	2	3	4	5	6	7	8	9	10	11
12	1	2	3	4	5	6	7	8	9	10	11	12

ASTRONAUT MATH

When the "modern" programs in elementary school mathematics were introduced to the classrooms, one of the more obvious components that was "new" was the inclusion of nondecimal numeration--number bases other than base ten. It was generally believed that exposing children to these bases would result in their having a more thorough understanding of base ten. Research has not substantiated such claims, however.

Should one assume that nondecimal numeration lacks importance in the elementary mathematics curriculum? Not at all! There is need, though, to put such activities in proper perspective. Number bases can provide a stimulating and enjoyable experience for children when handled as enrichment. The following activity can be applied, with minor modifications, to any grade level following grade one.

* * * * * *

The year is 2000 A.D. A team of six astronauts has just landed on the planet Mathos, a planet inhabited by beings who look much like those on Earth, except for their hands, each having but a thumb and one finger. The astronauts introduce themselves and are made welcome.

During their stay, they learn many things about life on Mathos and find that it differs from Earth in only one significant way, that being the Mathos number system. Whereas Earth has ten digits (0,1,2,3,4,5,6,7,8,9), Mathos has only four (○, ▽, □, △).

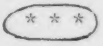
After a lengthy discussion, the astronauts are able to construct a chart illustrating the meaning of each digit and how each one is pronounced:

Number Represented	Written Digit	Pronunciation
	○	"Bong"
	▽	"Zip"
	□	"Crash"
	△	"Fizzle"

One must remember that the Mathos system has only four digits and that all other numbers are represented by combinations of those digits. On Earth, numbers are represented by combinations of our ten digits.

Counting on Mathos

Since Earthlings, when asked to count, begin with "one" rather than "zero," it should be noticed that counting in Mathosian would begin with "zip." The set below represents "fizzle" objects.



If one more object was placed in the set, there would be "zip-bong" of them. Do you see why? Mathos operates in a base four system of numeration, probably because they only have a total of four fingers and thumbs. By adding one more to the set as we did, that number represented one of the base with nothing remaining, therefore "zip-bong." A deeper understanding should result if Mathosian is compared to base four as it is written on Earth.

Set	Mathos	Earth
	▽	1
	□	2
	△	3
	▽ ○	10
	▽ ▽	11
	▽ □	12
	▽ △	13
	□ ○	20
	□ ▽	21
	□ □	22
	□ △	23
	△ ○	30
	△ ▽	31
	△ □	32
	△ △	33
	▽ ○ ○	100
	▽ ○ ▽	101

. . . and so on

* * * * * *

The reason this activity is liked by children is that once they have mastered the pattern, counting with the words ("zip - bong - crash," etc.) is both fun to do and funny to hear. Generally, the counting activity is sufficient. There may be times when the teacher will want the pupils to do some computation, e.g. $\square \triangle + \triangledown \bigcirc \triangle = ?$, but this should be reserved for only the above average students.

For a variation to the Mathosian counting, allow the children to supply their own words and symbols and work in bases other than base four. For a real switch, work in base twenty-six, using all the letters of the alphabet.

FACTORS, MULTIPLES, AND RELATED PATTERNS

Factors

When two numbers are multiplied, each of those numbers is a factor of the product. In the number sentence 3 X 8 = 24, 3 and 8 are factors of 24. If the equation read 24 ÷ 8 = 3, 3 and 8 are still considered factors.

Using this understanding of the term "factor," one can develop some "divisibility rules" for most of the whole numbers 2 - 9. One number, N, is divisible by another number, P, if P is a factor of N. Thus 12 is divisible by 2, 3, 4, and 6 because each of those numbers is a factor of 12.

Divisibility by 2

Divisibility by 2 is generally learned quickly by elementary pupils, because they realize that 2 is a factor of every even number. Thus, any number ending in 0, 2, 4, 6, or 8 is divisible by 2.

Divisibility by 3

Examine the numerals in sets A and B below. All in set A are divisible by 3. None in set B is. Can you see why?

Set A	Set B
3813	2413
2430	4310
9012	5302
3201	2941
2139	1259

Examining set A more closely, one will realize that 3 is a factor of the sum of the digits in each numeral.

3813 ⟶ 3 + 8 + 1 + 3 = 15

2430 ⟶ 2 + 4 + 3 + 0 = 9

9012 ⟶ 9 + 0 + 1 + 2 = 12

3201 ⟶ 3 + 2 + 0 + 1 = 6

2139 ⟶ 2 + 1 + 3 + 9 = 15

In set b, summing the digits of each numeral will result in a number not divisible by 3; therefore the number itself is not divisible by 3, as in 2413. 2 + 4 + 1 + 3 = 10. 3 is not a factor of 10. Therefore, 2413 is not divisible by 3.

Divisibility by 6

2 and 3 are factors of 6. A number is divisible by 6 if it is divisible by 2 and 3. Therefore, it must be an even number and the sum of its digits must be divisible by 3. Which of the following numerals is divisible by 6?

(a) 4,323 (d) 9,876

(b) 4,824 (e) 5,550

(c) 2,468

If you chose (b), (d), and (e), you're correct. 3 is a factor of (a), but 2 is not. 2 is a factor of (c), but 3 is not.

Divisibility by 9

In determining divisibility by 3, the digits of the numeral were added to see if 3 was a factor of that sum. The same process applies to divisibility by 9, except the sum has to have 9 as a factor. The

three numerals below are all divisible by 9.

$24111 \longrightarrow 2 + 4 + 1 + 1 + 1 = 9$

$89172 \longrightarrow 8 + 9 + 1 + 7 + 2 = 27$

$207 \longrightarrow 2 + 0 + 7 = 9$

Divisibility by 5

This rule, as with divisibility by 2, is quickly grasped by children. Any number ending in 0 or 5 has 5 as a factor, thus is divisible by 5.

Multiples

A multiple of a number is the product of any whole number and that number. For example, 0, 4, 8, 12, 16, etc., are all multiples of 4, because they are the products of 4 X 0, 4 X 1, 4 X 2, 4 X 3, 4 X 4, etc.

Multiples of 9

9 X 1 = 9

9 X 2 = 18

9 X 3 = 27

9 X 4 = 36

9 X 5 = 45

9 X 6 = 54

9 X 7 = 63

9 X 8 = 72

9 X 9 = 81

9 X 10 = 90

Through examination of the multiples for 9 above, a number of patterns should emerge:

(a) beginning at the top of the one's column and proceeding downward, the digits decrease by one each time.

(b) beginning at the bottom of the ten's place and going up has the same result of decreasing by one.

(c) the sum of the digits in each product is equal to 9.

Multiples of 7

7
14
21
28
35
42
49
56
63
70
77
etc.

There are three patterns resulting from the multiples of 7. The first involves only the one's digit. One will notice that each of the

ten digits (0-9) appears only once in the first ten multiples shown and then the same order of those digits will repeat. Also, the order of the successive one's digits has a unique pattern--

-3, -3, +7, -3, -3, +7, etc. as in:

7 (-3) = 4

4 (-3) = 1

1 (+7) = 8

8 (-3) = 5

5 (-3) = 2

2 (+7) = 9

etc.

The other pattern involves the sum of the digits, continuing to sum until the result is a single digit:

$7 \longrightarrow 7$

$14 \longrightarrow 1 + 4 = 5$

$21 \longrightarrow 2 + 1 = 3$

$28 \longrightarrow 2 + 8 = 10 \qquad 1 + 0 = 1$

$35 \longrightarrow 3 + 5 = 8$

$42 \longrightarrow 4 + 2 = 6$

$49 \longrightarrow 4 + 9 = 13 \qquad 1 + 3 = 4$

$56 \longrightarrow 5 + 6 = 11 \qquad 1 + 1 = 2$

$63 \longrightarrow 6 + 3 = 9$

$70 \longrightarrow 7 + 0 = 7$

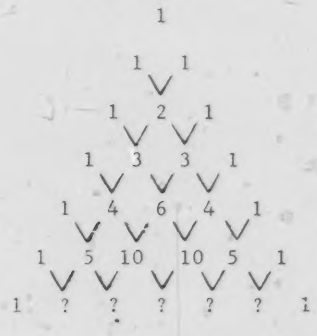
$77 \longrightarrow 7 + 7 = 14 \qquad 1 + 4 = 5$

The resulting sums, in order, illustrate a pattern of -2, -2, -2, +7, -2, -2, -2, +7, -2, -2, -2, +7, etc.

Some of the patterns and concepts discussed in this section can be of real value to a child in his everyday study of mathematics in school. Others may seem trivial, but they also have value in that they encourage mathematical thinking, an important ingredient in any good program of instruction.

PASCAL'S TRIANGLE

The 17th century French mathematician, Blaise Pascal, is given credit for this multi-faceted pattern, although many believe the Chinese were using it hundreds of years earlier. Examine the arrows (⋁) for assistance in determining how the triangle is constructed.



level 1 (sum=1)

level 2 (sum=2)

level 3 (sum=4)

level 4 (sum=8)

level 5 (sum=16)

level 6 (sum=32)

level 7 (sum=64)

If the triangle were extended through levels 8 and 9, what would the sums of the digits be?

Pascal's Triangle also has some application to probability for the elementary school mathematics curriculum, specifically as it relates to the flip of one or more coins, where each coin yields either a "head" or a "tail."

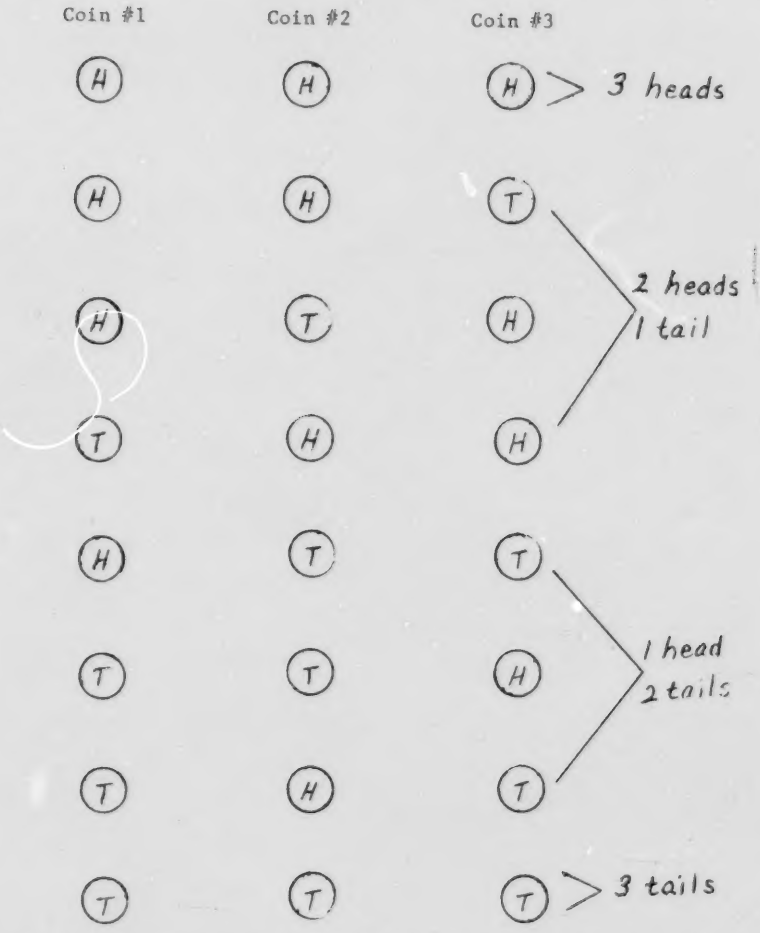
If one flips one coin, he has only two possibilities--a head or a tail, as illustrated below:

With Pascal's Triangle, this is shown in level 2.

Given two coins, the possibilities are increased from two to four.

Notice there is one chance in four that two heads will result, two chances of a head-tail combination, and one chance of getting two tails. This is demonstrated in level 3 as a 1-2-1 situation.

Level 4 indicates a 1-3-3-1 probability situation, meaning that when three coins are used, only one chance exists for three heads, or three tails, in every eight flips of the coins. Also, there are three chances for each situation yielding either 2 heads and 1 tail or 1 head and 2 tails. To verify this, we need only to diagram the possible outcomes.



The energetic teacher is encouraged to work with her pupils in diagramming the possibilities involving four or more coins. At the time of this writing, the authors were tired and decided to go to their respective homes for some sleep.

WHAT COMES NEXT?

It is a rare teacher who plans lessons to such a degree that the class period is over at precisely the time it should be. When too much is planned, the activity is carried over into the next period. When too little is planned, or the children work and learn at a faster than usual rate, the teacher is faced with a minor crisis--what to do with those few extra minutes.

Total group activities involving patterns will be an enjoyable activity for all children and at the same time can solve the teacher's problem.

The following series of patterns will span the entire elementary grades. The ingenious teacher can adapt them to any level of development.

A. △ ○ △ ○ △ ○ . . .

B. □ ▭ ▭ □ ▭ . . .

C. △ ○ □ □ △ ○ □ . . .

D. ↑ → ↓ ← ↑ → . . .

E. | L ⊔ □ | L . . .

F. 0, 2, 4, 6, 8, . . .

G. a, b, a, c, a, d, a, . . .

H. 0, 1, 3, 6, 10, 15, . . .

I. 1, 1, 2, 3, 5, 8, 13, . . .

J. 0, 2, 1, 3, 2, 4, 3, . . .

K. 1, 2, 5, 14, 41, . . .

L. 1, 2, 4, 8, 16, . . .

M. 2, 3, 5, 7, 11, 13, 17, 19, 23, . . .

N. C, T, T, F, F, S, . . .

O. J, F, M, A, M, J, J, A, . . .

"N" and "O" above are based on the number names (one, two, three, four, etc.) and the months of the calendar year (January, February, March, etc.) rather than a specific mathematical progression. Therefore, these should be used after the others. If not, the children will start looking for "tricks" in the patterns and the true benefit of the activity will be lost.

In addition to the type of patterns illustrated above, numerous other number relationships will prove interesting to children. Examine the following:

$$1 \times 9 + 2 = 11$$

$$12 \times 9 + 3 = 111$$

$$123 \times 9 + 4 = 1{,}111$$

$$1234 \times 9 + 5 = 11{,}111$$

etc.

If you see the pattern, determine if it continues. If so, how far?

Now, try another one.

$$1 \times 8 + 1 =$$

$$12 \times 8 + 2 =$$

$$123 \times 8 + 3 =$$

$$1234 \times 8 + 4 =$$

ALL ABOUT ELEVEN

It would certainly be possible to use these activities with elevens as types of drill. The teacher will, however, probably choose to let the pupils work through them as "just for fun" problems.

1. Look at the following number sentences.

$1 \times 11 = 11$
$2 \times 11 = 22$
$3 \times 11 = 33$
$4 \times 11 = 44$

Is there a pattern evident in the above which could be used to predict the product of 5 and 11; 6 and 11; 7 and 11; 8 and 11; and 9 and 11?

2. Is it possible to find a pattern for the product of 11 and another two-digit number?

$23 \times 11 = 253$
$45 \times 11 = 495$
$36 \times 11 = 396$
$52 \times 11 = 572$
$42 \times 11 = \ ?$
$27 \times 11 = \ ?$

Pupils can often quite readily observe that in each of the first four number sentences above that the digit in ten's place of the product is the sum of the digits of the factor other than 11. In other words:

$2 + 3 = 5$ (5 is the ten's digit of the product)
$4 + 5 = 9$ (9 is the ten's digit of the product)
$3 + 6 = 9$ (9 is the ten's digit of the product)
$5 + 2 = 7$ (7 is the ten's digit of the product)

Since $4 + 2 = 6$, $42 \times 11 = 462$.
Since $2 + 7 = 9$, $27 \times 11 = 297$.

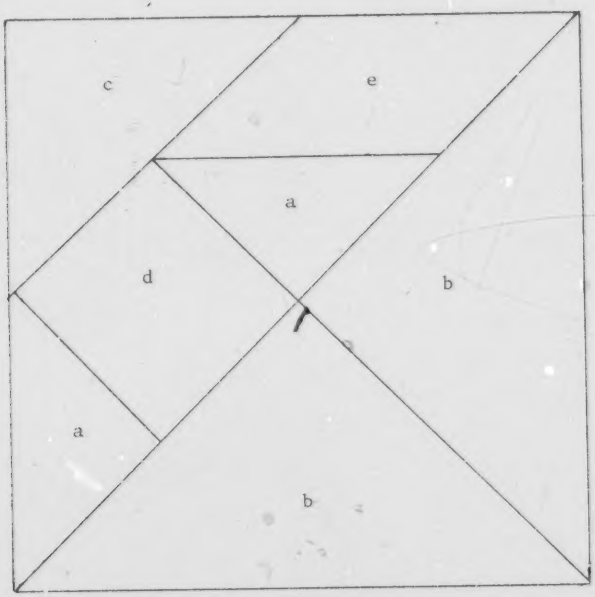
Note that in the case of examples such as 67×11, when the sum of the digits of the factor other than 11 is ten or more, it is necessary to regroup from ten's to hundred's place. Therefore:

$67 \times 11 = 737$ (6 + 7 = 13. The ten's digit of the product becomes 3; the 10 tens are renamed as 1 hundred.)

THE TANGRAM

Some children will enjoy examining this classic puzzle of Chinese origin, realizing many of its mathematical properties.

While there are many ways in which a square may be dissected into the seven pieces, called tans, the following is thought to be the original tangram.



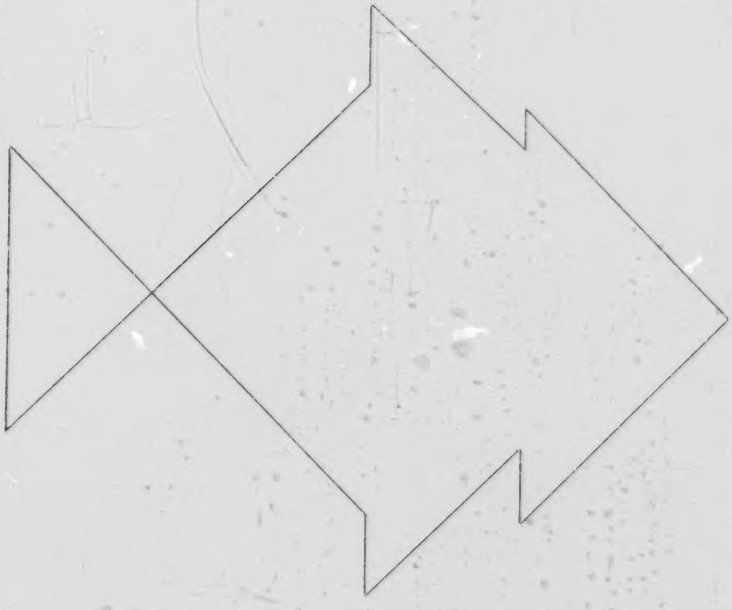
Pupils may find it interesting to cut carefully along the lines and then investigate some of the ways in which the pieces are related. For example:

1. What could be said about the relationship of the two small triangular shapes (a) to the square shape (d)?
2. How are the shapes marked "a" and the shape marked "c" related?
3. How are the "a" shapes and the "b" shapes related?
4. How are the "b" shapes and the "c" shape related?

Another extension could include considering the small triangular shapes (a) as units. What, then, would be the area of:

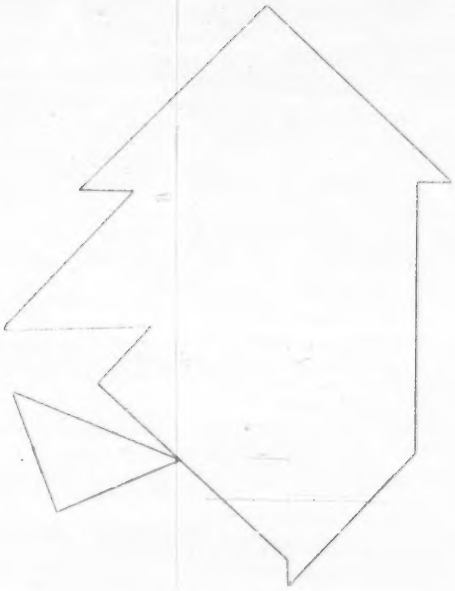
1. shape "c"?
2. shape "b"?
3. shape "d"?
4. shape "e"?
5. the entire tangram?

Tangrams have mainly been used by taking the tans and arranging them in interesting shapes. It is important that when building the shapes that all seven tans be used for each shape and that no tans overlap. While the more classic shapes derived from tangrams are "people figures," the shapes may take the form of animals or whatever the ingenuity of the creator of the puzzle permits, staying within the guidelines listed above. The following are but a few of the shapes which may be constructed. Elementary pupils will enjoy making their own puzzle shapes and letting other members of the class try to solve them.



FISH

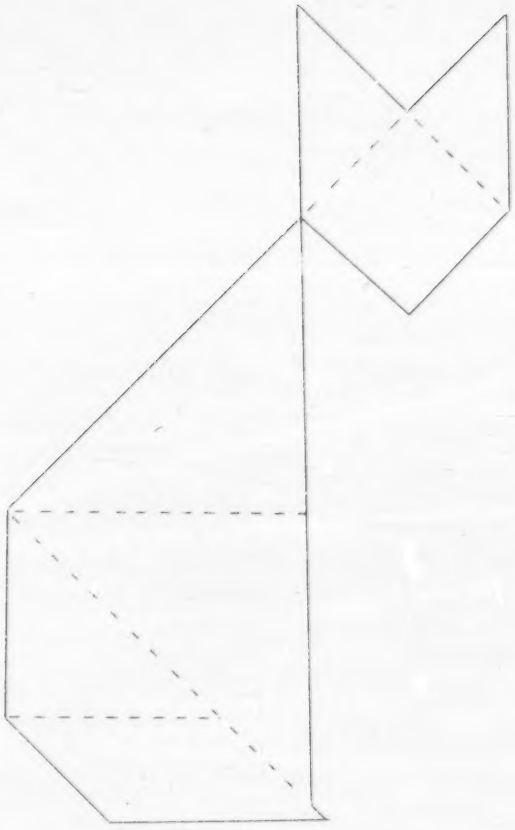
CAT



MAN

It should be noted that the figures shown here are not constructed to the same scale as the original square, so the solutions cannot be found by fitting the tans over the figures exactly. It is, rather, a matching of the shape that should be done in solving each puzzle.

On the following page an illustration of the solution to the "cat" puzzle is given, with dotted lines used to illustrate the placement of the tans. The reader is encouraged to solve the other puzzles as well as develop other puzzles.



CAT (Solution)

JUST FOR FUN

The following problems may be best suited as enrichment activities for better pupils. It may be found, however, that all of the pupils will enjoy attempting the problems, even though the solutions may come from only a few pupils. Often, less able pupils can profit from an arrangement in which they are a part of a team to solve the problems, even though their contributions may be less than that of the more able pupils.

1. A hunter walks due south 8 miles from a given point. After then walking due east for another 8 miles, he shoots a bear. He drags the bear 8 miles due north from the spot where he shot it and finds that he is back to his original starting point. What color is the bear?

2. A man walked into Atkins' Appliance Store and purchased a $24 radio, paying for it with two $20 bills. Mr. Atkins did not have enough change, so he went next door to Rogers' Poultry Shop to get change for one of the bills.

Soon after the customer left with his change, Mr. Rogers ran into the store and angrily announced that the bill he had changed was counterfeit. Atkins reluctantly gave Rogers the other $20 bill, which was not counterfeit. How much did each man lose?

3. Three hikers, Able, Baker, and Carter met on a mountain trail. They decided to divide their supplies equally among themselves and hike together. Each man had $10.

Able and Baker each has 3 loaves of bread, while Carter had none. Baker and Carter each had 3 pounds of meat, while Able had none. The men agreed that each would pay for what he received.

If 2 loaves of bread and a pound of meat together cost $1, and if a loaf of bread costs half as much as a pound of meat, how much money did each man have after the supplies were divided?

4. In Fairland, Indiana, 5 policemen can write 5 parking tickets in 5 minutes. The entire police force can write 100 tickets in one hour and 40 minutes. How many men are there on the police force?

5. A man has 10 pairs of blue socks, 6 pairs of green socks, and one pair of yellow socks. The socks are placed in a drawer unpaired. If the man had to find a pair of socks in the dark, how many socks would he have to take from the drawer to be certain of having a matching pair?

6. Marbles come in 10 different colors. There are 15 marbles in a package. How many packages must you buy to be sure of having at least 10 of one color?

* * * * * *

Solutions: Just for Fun

1. The bear was white, since the conditions of the problem indicate that the hunter was at the North Pole.

2. This puzzle can be solved by observing how the money and the goods changed hands. Mr. Rogers neither gained nor lost, since he paid out $20 and received a good $20 bill. Because the counterfeit bill had no value, the customer came to the appliance store with $20 and left with a radio costing $24 and $16 in change. Therefore, he gained $20. Since Mr. Rogers neither gained nor lost, the money gained by the customer must have been lost by Mr. Atkins. He lost $20.

3. If a loaf of bread costs half as much as a pound of meat, and since two loaves of bread and a pound of meat costs $1, a loaf of bread must cost 25¢ and a pound of meat 50¢. Baker gave Carter a loaf of bread and gave Able a pound of meat, so he received 75¢. Carter gave Able a pound of meat and received a loaf of bread from each of the others. Therefore, he received 50¢ and then paid out 50¢. Able received a pound of meat from Carter and a pound from Baker. He gave Carter a loaf of bread. Therefore, he received 25¢ and paid out $1. Able had $9.25; Baker had $10.75; Carter had $10.00.

4. If 5 men can write 5 tickets in 5 minutes, one man can write 1 ticket in 5 minutes, and he can write 20 tickets in 100 minutes. Since 100 tickets are written in 100 minutes, 5 men are needed. Therefore, there are 5 men on the force.

5. 4 socks. After 3 socks have been drawn, the man may have a pair of the same color, but he may have only one of each color. The fourth sock that is drawn is certain to be a mate of one of the first 3 drawn. Therefore, at least 4 socks must be drawn to be <u>certain</u> of having a pair.

6. 7 packages. It is possible to have as many as 90 marbles without having 10 or any one color, for the 90 may be made up of 9 of each of the 10 colors. The ninety-first marble, then, would have to be the tenth of one color. Six packages contain 90 marbles, so it would take at least 7 packages to have 91 marbles and be certain of having 10 of one color.

COMPUTER PUNCH CARDS

When other forms of drill concerning the basic facts for addition have become a bore, elementary pupils may enjoy an activity in which they are pretending that they are constructing punch cards which are to be fed into a computer.

The teacher may choose to copy the "cards" to be used on the chalkboard, hand them out as dittoed sheets, or actually write them on file cards. A sample card could take the following form:

64	32	16	8	4	2	1	
				x	x	x	7
	x	x			x		50
x	x	x		x	x		118
		x	x	x	x	x	31

There are, obviously, many possibilities for this activity. In each case, the student's attention is directed to the fact that the numeral appearing at the far right represents a sum. The student's task then becomes one of using the numerals which appear at the top of the columns as addends in such a way that they may be added to get the specified sum. Each addend may be used only once for each sum given. In the first case shown above, the student needs to punch out the spaces corresponding to the numerals 4, 2, and 1, since $4 + 2 + 1 = 7$.

Like many activities of this type, this one is open-ended with respect to possible uses. The teacher may, for example, like to investigate the possibility of converting the punch cards in a way for use with subtraction, multiplication, or division.

BASIC FACT PRACTICE

As a review, or simply as additional practice periodically, the teacher may choose to use the following activity which can involve the practicing of the basic facts for addition and multiplication. Inspection of the activity will reveal the need for applying subtraction and division also.

Construct a partially completed table like the following:

Sum		12		17	20	24
Addend/Factor	7	5				
Addend/Factor	6		7			
Product			56	72	36	140

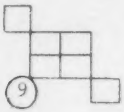
The nature of the solution is dependent upon what information is given within each column. In the first column, since the numerals 7 and 6 are supplied, the child is asked to think of the two as (1) addends, so the sum to be filled in is 13; and (2) factors, so the product to be filled in is 56.

Since the second column supplies a sum and an addend, the student will need to subtract to find the missing addend; then multiply the two factors to arrive at the product.

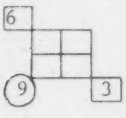
The possibilities for an activity such as this are limited only by the teacher's desire with respect to the kind of practice necessary at a given point in time.

MINI-TABLES FOR ADDITION

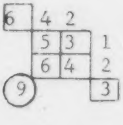
As a good practice for addition facts, the mini-table can be used for addition. Note in the following example that the number placed within the circle should represent a sum.



Once a particular sum has been chosen, as with the 9 in this case, let the pupils choose two addends of 9. These addends should then be placed within the boxes as:



After the two addends have been renamed (in this case, $6 = 4 + 2$ and $3 = 1 + 2$), let the pupils carry out the indicated additions so that the inside of the table has been completed. These operations have been completed in the following.



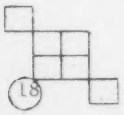
Did the pupils notice that the sum of either of the diagonals ($5 + 4$ or $6 + 3$) is equal to the original sum?

As with the mini-tables for multiplication, the teacher may wish to vary the format of the mini-tables for addition according to the amount of information which is supplied at the beginning. The pupils may also wish to begin with a completely empty table and build their own.

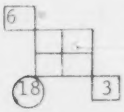
The teacher may decide to let some of the more able upper-grade pupils, following work with the integers, rename the sum of 9, for example, as 12 and -3, 14 and -5, etc. and then complete the remainder of the table.

MINI-TABLES FOR MULTIPLICATION

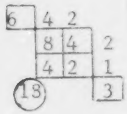
As an activity that can be used with individuals, small groups, or with the entire class, the teacher might wish to consider these "mini-tables." Note the following example:



In this case, if 18 is considered to be a product, the pupils should decide upon two factors of 18 which could be placed in the two boxes in the following manner:



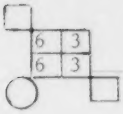
The table may now be completed, using the distributive property. The 6, for example, may be renamed as 4 + 2 and the 3 renamed as 2 + 1.

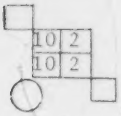


Note that the numerals inside the table are derived by performing the usual multiplications. That is, 4 X 2 = 8; 4 X 1 = 4; 2 X 2 = 4; and 2 X 1 = 2.

Check to see if the pupils noticed that the sum of the numerals found within the table (8, 4, 4, and 2) is equal to the product (18).

The teacher will note many possibilities for varying this activity, according to the information supplied originally in the table, which two factors are chosen, etc. It may be decided, for example, that an interesting variation would be to supply some of the numerals within the table and let the pupils use those to find the factors and the product. Examples of this type would include:



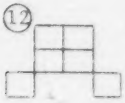


For the first example above, the product would be 18 and the factors 9 (renamed as 6 + 3) and 2 (renamed as 1 + 1). In the second example, the product would be 24 and the factors 6 (renamed as 5 + 1) and 4 (renamed as 2 + 2).

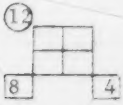
The teacher may decide to use these table purely for purposes of enrichment. They do, however, provide a rather novel means for practicing the facts.

MINI-TABLES FOR SUBTRACTION

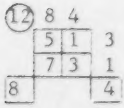
While the mini-tables do not lend themselves very readily to subtraction, a variation of them can be used if the teacher feels that the difference will not lead to confusion when using the tables for addition and multiplication, and if the numbers to be used are chosen carefully. Note in the following example that the sum is still to be placed within the circle, but the circle is positioned in a different place than was true for the addition or multiplication example.



The sum can now be separated into two addends (in this case, 8 and 4).

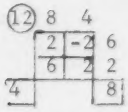


After 12 has been renamed as 8 + 4 and 4 has been renamed as 3 + 1, the indicated subtractions can be carried out.



Since the sum of the diagonals is equal to the one addend which was not renamed (8), the teacher may wish to leave the box for that addend empty until the table has been completed.

Mention was made above that the numbers to be used must be chosen carefully. Consider the mini-table below in which the position of the two addends, 8 and 4, have been reversed.

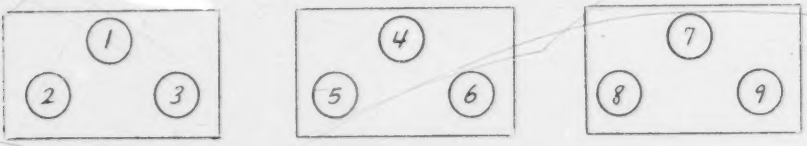


Note what can now happen after the renaming and the carrying out of the indicated subtractions. Obviously, if the pupils have not had full exposure to the integers, this table, with the use of signed numbers, will cause problems.

IT'S YOUR MOVE

For this activity, it may be helpful for the pupils to actually cut the shapes out of construction paper so that they may be easily moved back and forth.

Here are three rectangles, each one containing three numerals. Move one numeral from one of the rectangles to one of the other rectangles so that the sum of the numerals in each rectangle is equal to the sum of the numerals in each of the other rectangles.



Is it possible to construct other examples similar to this? Could the pupils develop some?

WHAT'S THE SIGN?

This will prove to be an interesting "think and drill" activity for many elementary pupils. The teacher may want to ditto the exercises or let the children copy them after they have been written on the chalkboard. Be sure that the pupils understand that in each activity there are number sentences written horizontally and vertically.

In each of the following, some of the numerals are arranged like a number sentence to produce a given result. The arithmetical signs of operations are missing, however. The task is to try to fill in the correct signs. It is possible that some of the numerals are a part of the set of negative integers.

1		6		2	= 3
9		4		8	= 5
5		3		2	= 4
=		=		=	=
2		8		5	= 11

2		8		4	= 14
6		8		3	= 16
4		1		7	= 11
=		=		=	=
3		1		5	= 9

2		8		4	= 4
6		8		3	= 16
8		7		3	= 5
=		=		=	=
20		9		4	= 15

Using frames like the empty one above, the teacher and pupils will probably want to develop other examples of this type of activity. Are there ways to structure this activity to use numbers other than whole numbers?

PALINDROMIC NUMERALS

The term palindrome is usually thought of with respect to words or phrases which are the same whether they are read forward or backward. "Mom," "toot," and "noon" would, therefore, be examples of palindromes. Numerals may also be palindromic in nature. Examples could include 11, 232, 6776, etc.

For pupils who might enjoy a means for producing palindromic numerals, the following procedure is recommended.

Given a numeral such as 43. Reverse the digits and add:

$$\begin{array}{r} 43 \\ +34 \\ \hline 77 \end{array}$$

Other easy examples could include:

$$\begin{array}{r} 52 \\ +25 \\ \hline 77 \end{array} \qquad \begin{array}{r} 56 \\ +65 \\ \hline 121 \end{array}$$

In many instances, the reversing of the digits and adding does not produce a palindromic numeral immediately. The procedure may need to be repeated as:

$$\begin{array}{r} 387 \\ +783 \\ \hline 1170 \end{array} \qquad \begin{array}{r} 1170 \\ +\ 711 \\ \hline 1881 \end{array}$$

$$\begin{array}{r} 37 \\ +73 \\ \hline 110 \end{array} \qquad \begin{array}{r} 110 \\ +\ 11 \\ \hline 121 \end{array}$$

$$\begin{array}{r} 79 \\ +97 \\ \hline 176 \end{array} \qquad \begin{array}{r} 176 \\ +671 \\ \hline 847 \end{array} \qquad \begin{array}{r} 847 \\ +748 \\ \hline 1595 \end{array} \qquad \begin{array}{r} 1595 \\ +5951 \\ \hline 7546 \end{array} \qquad \begin{array}{r} 7546 \\ +6457 \\ \hline 14003 \end{array} \qquad \begin{array}{r} 14003 \\ +30041 \\ \hline 44044 \end{array}$$

Through their own experiences, children can discover some interesting patterns in constructing palindromic numerals. As with the last example, it can also serve as a purposeful practice for addition facts.

TABULAR PATTERNS

When there are pupils in the elementary classroom who really need additional drill, but are being "turned off" by flash cards and similar devices, the teacher can often find a different way to accomplish the purpose. A number of activities like the following can often provide just the kind of variety needed.

1. Look at the following series of numerals. Is there a pattern between the numerals in Column A and those in Column B?

A	B
2	5
3	6
4	7
5	8
.	.
.	.
.	.

What would the next A and B numerals be?

2. Complete the B column in the following series:

A	B
2	7
5	16
4	
10	

What is the pattern? If the table was extended, what would the next A and B numerals be?

* * * * * *

Solutions

1. The pattern can be continued by adding 3 to each numeral in the A column (A + 3 = B).

2. The pattern can be continued by multiplying each numeral in the A column by 3 and then adding 1 (3A + 1 = B).

Activities like this can bring about a welcome change as a drill activity. Obviously, the variety of exercises such as this is limited only by the teacher's imagination and the level of maturity of the pupils.

FIND THE MISSING DIGITS

For this activity the pupils should be encouraged to find as many solutions as they can. They may then enjoy comparing solution sets with each other.

Let the children copy the problems onto another sheet of paper. Help them to understand that each dot (·) represents a missing single-digit number within the set of whole numbers. The task is to figure out what the missing numbers are. Each dot may represent a different number, but the substitution must in each case be a single-digit number.

1. $\begin{array}{r} \cdot \\ +6 \\ \hline \cdot \\ \cdot \end{array}$

2. $\begin{array}{r} \cdot \\ +\cdot \\ \hline \cdot \\ 5 \end{array}$

3. $\begin{array}{r} \cdot \\ +7 \\ \hline \cdot \\ \cdot\cdot \end{array}$

4. $\begin{array}{r} \cdot \\ +8 \\ \hline \cdot \\ 22 \end{array}$

Could the pupils think of other examples of this type? Could the problems be adapted for use with subtraction?

The pupils will find this a novel and interesting way to practice the facts.

* * * * * *

Solution

1. $\begin{array}{rrr} 1 & 1 & 2 \\ +6 & +6 & +6 \\ \hline +1 & +2 & +1 \\ \hline 8 & 9 & 9 \end{array}$

2. $\begin{array}{rrrrrr} 1 & 1 & 1 & 2 & 2 & 3 \\ +1 & +2 & +3 & +1 & +2 & +1 \\ \hline +3 & +2 & +1 & +2 & +1 & +1 \\ \hline 5 & 5 & 5 & 5 & 5 & 5 \end{array}$

3. $\begin{array}{rrrrrrrr} 1 & 2 & 4 & 5 & 6 & 7 & 8 & 9 \\ +7 & +7 & +7 & +7 & +7 & +7 & +7 & +7 \\ \hline +2 & +1 & +9 & +8 & +7 & +6 & +5 & +4 \\ \hline 10 & 10 & 20 & 20 & 20 & 20 & 20 & 20 \end{array}$

4. $\begin{array}{rrrrr} 5 & 6 & 7 & 8 & 9 \\ +8 & +8 & +8 & +8 & +8 \\ \hline +9 & +8 & +7 & +6 & +5 \\ \hline 22 & 22 & 22 & 22 & 22 \end{array}$

CASTING OUT NINES

As an interesting enrichment activity, the idea of casting out nines can be an effective means for checking computations. While the explanations may appear lengthy and detailed, working through a few examples reveals the real simplicity of the idea.

For Addition. Consider the work completed on the following example:

$$\begin{array}{r} 352 \\ 794 \\ +214 \\ \hline 1360 \end{array}$$

The accuracy of this computation can be checked by casting out nines in the following manner:

352	(3+5+2=10. Subtracting all nines possible leaves 1.)
794	(7+9+4=20. Subtracting all nines possible leaves 2.)
+214	(2+1+4= 7. Subtracting all nines possible leaves 7.)
1360	(1+3+6+0=10. Subtracting all nines possible leaves 1.)

Note in this example that after nines have been cast out of the three addends the numerals remaining are 1, 2, and 7. Compute 1 + 2 + 7 = 10. If all the nines possible are then subtracted from this value (10), a 1 remains. Comparison reveals that 1 was also left when all nines possible were cast out of the sum. The answer checks.

For Subtraction. While not as straightforward, examination of the following will reveal the check for subtraction by casting out nines:

386	(The sum of 3, 8, and 6 is 17.)
-195	(1+9+5=15. Subtracting all nines possible leaves 6.)
191	(1+9+1=11. Subtracting all nines possible leaves 2.)

Note that 17 - 6 = 11. If all of the nines possible are subtracted from 11, the remaining 2 is the same as the remainder when all nines possible were removed from the answer of the computation. The answer checks.

For Multiplication.

357	(3+5+7=15. Subtracting all nines possible leaves 6.)
X 48	(4+8=12. Subtracting all nines possible leaves 3.)
2856	
1428	
17136	(1+7+1+3+6=18. Subtracting all nines possible leaves 0.)

Take the two remainders from the two factors that have been cast out (6 and 3). The product of these is 18. If all of the nines

possible are subtracted from 18, the result is 0; the same result as obtained when all nines were cast out of the product. The answer checks.

For Division.

$$17\overline{)204}\quad 12$$

(1+2=3; 1+7=8. 3 X 8 = 24)
(2+0+4=6)

After summing the digits of the divisor and the quotient, find the product of the two numbers obtained (2 X 8 = 24). If all of the nines possible are then subtracted from 24, 6 is left. This corresponds to the result of casting out nines from the dividend (2 + 0 + 4 = 6). The answer checks.

INTUITIVE MEASUREMENT

The following activities might be used as preliminary discussions prior to a more formal study of various aspects of measurement.

1. Suppose you had two sticks. One stick is one centimeter long; one is one meter long. Which stick would you use to measure:

   a. the width of your hand? Why?
   b. the width of the classroom? Why?

2. Suppose you had a balance scale, a set of hectogram weights, and a set of gram weights. Which would you use to balance the weight of:

   a. ten small candies? Why?
   b. ten large apples? Why?

3. What is the best answer for each of the following?

   a. How long is a sheet of notebook paper?
      1. about 3 centimeters long.
      2. about 3 dekameters long.
      3. about 3 meters long.
   b. How high is the door of the classroom?
      1. about 3 centimeters high.
      2. about 3 dekameters high.
      3. about 3 meters high.

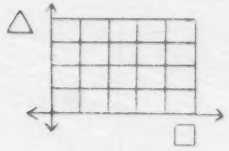
4. Choose the best answer for each of the following.

   a. How much might an apple weigh?
      1. about 170 grams.
      2. about 170 kilograms.
   b. How much might a cat weigh?
      1. about 3 grams.
      2. about 3 kilograms.
   c. A third-grade boy might weigh how much?
      1. about 30 grams.
      2. about 30 kilograms.

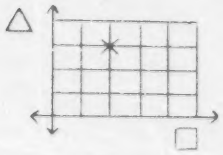
TIC-TAC-TOE

This is a game that can be played and enjoyed by the entire class. After the fundamentals of the game are mastered, pairs of students will enjoy playing it on a one-against-one basis.

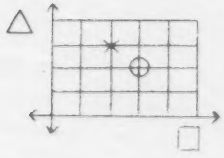
After the class has been divided into two teams (one team can be the "X" team; the other the "O" team), explain that the object of the game is for either team to place four X's or O's in a row, either horizontally, vertically, or diagonally on the game board. The game board can be constructed by the teacher on the chalkboard or on a transparency for use with an overhead projector.



The first player of the team chosen to go first makes a move by supplying an ordered pair of numbers. If the player chooses (2,3), for example, note that the first numeral <u>always</u> denotes movement along the □ line, while the second numeral <u>always</u> denotes movement along the △ line. Such a move would be placed on the game board as:



The first player on the opposing team may then decide to move (3,2). The board would then appear as:



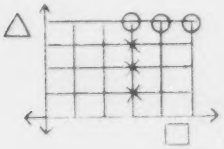
To keep pupils from selecting ordered pairs which have already been used, a table of moves such as the following may be kept. Following further experience with the game, such a table will probably not be necessary.

□	△
2	3
3	2
.	.
.	.
.	.
.	.

The game is continued, alternating between teams, until one team has "four in a row."

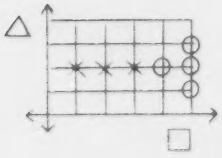
There are two difficulties which are usually noticed with pupils in early use of the game. Some pupils would count the first "X" move, above, as (3,4). These children are, of course, beginning their counting at the base line, not yet understanding that that is zero.

In a similar manner, the pupil who wants to make a move for the "X" team to win the game may have difficulty with the following:



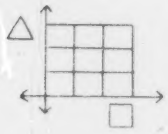
Practice will tell the child that the move he is seeking is (2,0), which says, "Go out 2 along the box line; go up none at all on the triangle line."

A similar problem would occur with the following:



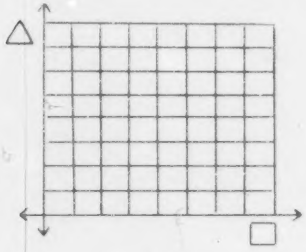
In this case, the best "X" move is (0,2), which says, "Go across none at all on the box line; go up 2 on the triangle line."

To structure the game in a way which will force the use of zero, use a game board of the type:



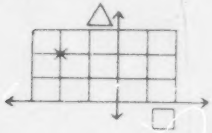
Now, of course, the zero must be used at some point in the game in order to get "four in a row."

As the game is used, students become quite proficient with regard to planning ahead to score or to block an opponent from scoring. The blocking skill, in fact, becomes so good that the size of the game board may need to be increased. It may take the form:



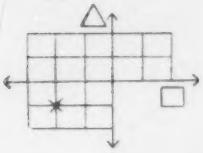
The board can, of course, be made larger.

Interesting and instructive variations or extensions of the game can include:

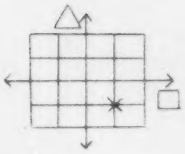


The ordered pair which is associated with this move would be (-2,2), or, "Go 2 spaces left of zero on the box line; go up 2 spaces on the triangle line." The order pair could be read "negative two, positive two," or, "negative two, two."

The game could be further extended to include:



In this case, the ordered pair for this move would be (-2,-1).



Here, the ordered pair would be (1,-1).

Try it! The children will like it!

BOX TIMES BOX

This is an activity that can be used for enrichment for individuals, small groups, or the entire class. Since the children are searching for two "secrets" which will enable them to complete the sentences properly, the teacher will probably want to caution those "early discoverers" not to give the secrets away.

Consider the following open sentence:

$$(\square \times \square) - (5 \times \square) + 6 = 0$$

The task here is to complete the sentence in such a way as to make the sentence true. Help the children remember that when a number is substituted for the box, each other box must be replaced by the same number. A solution will take the form:

$$\begin{aligned} (\boxed{3} \times \boxed{3}) - (5 \times \boxed{3}) + 6 &= 0 \\ 9 - 15 + 6 &= 0 \\ -6 + 6 &= 0 \\ 0 &= 0 \end{aligned}$$

OR

$$\begin{aligned} (\boxed{2} \times \boxed{2}) - (5 \times \boxed{2}) + 6 &= 0 \\ 4 - 10 + 6 &= 0 \\ -6 + 6 &= 0 \\ 0 &= 0 \end{aligned}$$

Note that the truth set for the open sentence contains two elements. In the above case, for example, both 3 and 2 could be substituted for the box and cause the sentence to be a true sentence. Remembering that there will always be two elements in the truth set of sentences of this type, observe the following additional examples:

$(\square \times \square) - (7 \times \square) + 12 = 0$ The truth set is $\{3,4\}$

$(\square \times \square) - (8 \times \square) + 12 = 0$ The truth set is $\{2,6\}$

$(\square \times \square) - (13 \times \square) + 12 = 0$ The truth set is $\{1,12\}$

Rather quickly children discover that the two elements of the truth set satisfy the following conditions:

1. The two elements are <u>addends</u> of the first numeral in the open sentence.

2. The two elements are <u>factors</u> of the second numeral in the open sentence.

Some children will discover one of the two conditions (secrets) without having discovered both of them. Experience with other open sentences of this type causes these pupils to realize that in order to arrive at the correct elements of the truth set, both conditions must be met.

Using this pattern it is rather easy for the teacher to construct many additional open sentences of this type. The pupils who have discovered both "secrets" may enjoy making up their own sentences to use with the rest of the group or with a friend during free time.

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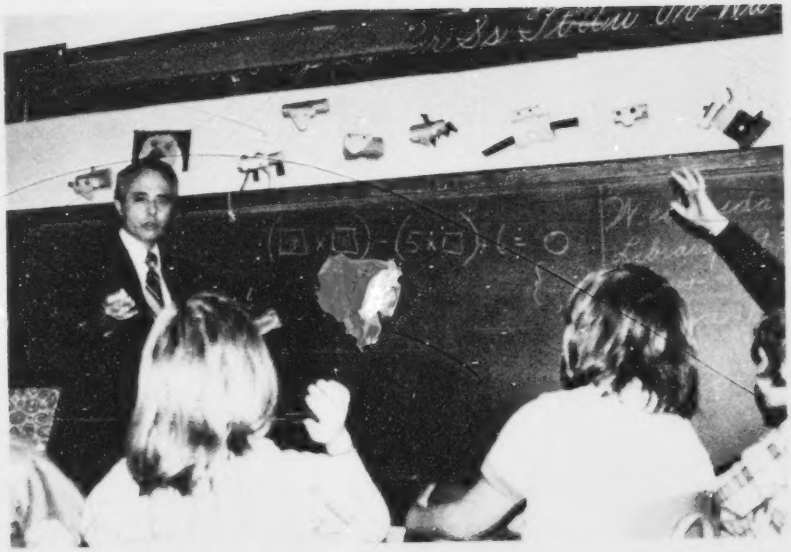
Periodicals

Rather than listing specific articles, the authors suggest that three journals be consulted. About every issue of The Arithmetic Teacher, Instructor, and Teacher (formerly Grade Teacher) contains activities appropriate to meaningful mathematics instruction in the elementary school classroom.



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